PRESSURE AND TEMPERATURE OF SUBTERRANEAN LIQUIDS AS EARTHQUAKE INDICATORS

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Much interesting experimental material has been accumulated [1-5] concerning fluid precursors of earthquates, in the form of hydrorheodynamic and geometric effects. It is well known [4] that underground waters react to elastic deformations of the earth's core caused by tidal forces, while periodic variations have been observed in the output of artesian wells. Such reactions of supply strata are of special significance in predicting earthquates, since earthquate initiation is always accompanied by changes in the stress–strain state of earth components.

Changes in the volume of a saturated rock mass produce an increase or decrease in the water table, and a well system in such strata with accurate level recorders forms an excellent seismograph.

The present study will utilize the theory of heterogeneous media to perform a precise analysis of the reactions of a stratum completely saturated by liquid to periodic long-wave perturbations of the stress state in the form of perturbations and interactions of the temperature and pore pressure fields. The amplitude–frequency characteristics obtained for the supply stratum may be used for prediction of earthquakes as well as tidal ebb and flow processes in the earth's core.

1. Assume a linearly elastic stratum completely saturated by liquid which is slightly compressible. Then for small perturbations of the pore pressure the continuity equations of the solid and liquid phases, the filtration law, and the equations of conservation of energy of each phase are linearized. If the temperatures of the phases are equal then these equations have the form [6]

$$\frac{\partial m}{\partial t}+\frac{\beta_1}{3}\frac{\partial\theta^f}{\partial t}-\beta_1(1-m_0)\frac{\partial P}{\partial t}+\alpha_1(1-m_0)\frac{\partial T}{\partial t}-(1-m_0)\frac{\partial \dot{u}_j}{\partial x_j}=0, \tag{1.1}$$

$$\frac{\partial m}{\partial t}+\beta_2 m_0\frac{\partial P}{\partial t}-\alpha_2\frac{\partial T}{\partial t}+m_0\frac{\partial v_j}{\partial x_j}=0, \tag{1.2}$$

$$\frac{\partial P}{\partial x_i}+\frac{\mu m_0}{k_0}(v_i-\dot{u}_i)+\rho_2^0\frac{\partial W}{\partial x_i}=0, \tag{1.3}$$

$$\frac{\partial T}{\partial t}-\frac{\alpha_2 T_0}{c_p}\frac{\partial P}{\partial t}=\frac{d_2}{c_p}\nabla^2 T, \tag{1.4}$$

$$\sigma_{ij}^f=2G\left(e_{ij}+\frac{\nu}{1-2\nu}e\delta_{ij}\right)+(\varepsilon P-(1-m_0)\alpha_1 KT)\delta_{ij}, \tag{1.5}$$

where

$$\theta^f=\sigma_{ij}^f\delta_{ij},\quad e=e_{ij}\delta_{ij},\quad \Gamma_{ij}=\sigma_{ij}^f-P\delta_{ij},\quad \varepsilon=(1-m_0)\beta_1 K,$$
$$e_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\quad \dot{u}_i=\frac{\partial u_i}{\partial t},\quad \delta_{ij}=\begin{cases}1, & i=j,\\ 0, & i\neq j.\end{cases}$$

Here u_i are solid particle displacements: v_i, liquid particle velocities; e_{ij} and σ_{ij}^f, components of the deformation and effective stress tensors; P, pore pressure perturbation; T, liquid phase temperature perturbation; ρ_1^0, ρ_2^0 and β_1, β_2, density and isothermal compressibility coefficient of solid and liquid phase; m, deviation of stratum porosity from initial value m_0; α_1, α_2, volume expansion coefficients; d_2, c_p, thermal conductivity and heat capacity (at constant pressure) of liquid phase, calculated per unit volume; k_0, μ, stratum permeability and liquid viscosity coefficients; ν, E, Poisson coefficient and Young's modulus; $(1-m_0)K$, compression modulus of stratum skeleton; W, potential of external forces; subscript 0 indicates unperturbed state.

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The process of earthquake initiation or tidal ebb/flow occurs slowly, so that the inertial force may be neglected in Eq. (1.3). From continuity equation (1.1) and Hooke's law, Eq. (1.5), we obtain

$$m = (1 - m_0)(1 - \beta_1 K)(e + \beta_1 P - \alpha_1 T),$$
$$e = (\Gamma_{hk} + 3P)/[3(1 - m_0)K - \beta_1 P + \alpha_1 T. \tag{1.6}$$

Substitution of Eqs. (1.3), (1.6) in the liquid phase continuity equation leads to the following expression

$$\frac{\partial P}{\partial t} + a\frac{\partial \Gamma_{hk}}{\partial t} + b\frac{\partial T}{\partial t} = \chi\nabla^2\left(P + \rho_2^0 W\right), \tag{1.7}$$

where

$$a = \frac{1-\varepsilon}{3(1-m_0)\beta K}, \quad \beta = m_0\beta_2 + \frac{1-(1+m_0)\varepsilon}{(1-m_0)K}, \quad \chi = \frac{k_0}{\mu\beta}, \quad b = (\alpha_1 - \alpha_2)\frac{\chi\mu m_0}{k_0}.$$

We assume that the well to be considered ($u_r(r_c, t) = 0$) with radius r_c is located in an infinite plane thermally insulated saturated stratum and that the stress field is perturbed by external forces. Flow of liquid and heat over a time Δt into a well of cross section S_1 changes the liquid level by an amount $\Delta\xi$, so that $\Delta V = S_1\Delta\xi$. This leads to a pressure change $\Delta P = \gamma\Delta\xi$, as a result of which thermal expansion of the liquid α_2 changes the liquid temperature in the well itself, as defined by the equation

$$\Delta T_1 = (\beta_2/\alpha_2)\Delta P. \tag{1.8}$$

It is evident that the compressibility β_2 leads to a change in volume equal to $-\beta_2\Delta P(V - \Delta V) \approx -\beta_2\gamma V\Delta\xi$. Then the condition of conservation of volume of an element is written in the form

$$S_1\Delta\xi = \Delta V_l + \Delta V_s - \beta_2 V\gamma\Delta\xi. \tag{1.9}$$

Moreover, for liquid filtration and heat flow in the porous medium we use Darcy's law Eq. (1.3), and Fourier's law [6]:

$$\Delta V_l = -S_2 V_r \Delta t = \frac{k_0 S_2}{\mu}\left(\frac{\partial\left(P + \rho_2^0 W\right)}{\partial r} - m_0\dot{u}_r\right)\Delta t,$$
$$\Delta Q = -S_2 q\Delta t = S_2 d_2\frac{\partial T}{\partial r}\Delta t, \quad \Delta Q = c_p V\Delta T,$$
$$\Delta V_s = \alpha_2 V\Delta T, \quad \Delta T = \frac{S_2 d_2}{c_p V}\frac{\partial T}{\partial r}\Delta t, \tag{1.10}$$

where

$$V_r = m_0 v_r, \; S_2 = 2\pi r_c h.$$

We substitute Eq. (1.10) in Eq. (1.9) and obtain the desired boundary condition on the well wall in simplified form

$$d\xi/dt = D\partial P/\partial r + B\partial T/\partial r + \Phi, \tag{1.11}$$

where

$$D = \frac{k_0 S_2}{\mu S_0}, \quad B = \frac{\alpha_2 d_2 S_2}{c_p S_0}, \quad S_0 = S_1 + \beta_2\gamma V, \quad \Phi = D\rho_2^0\frac{\partial W}{\partial r}, \quad \dot{u}_r(r_c, t) = 0.$$

The secondary boundary conditions will be

$$P = \gamma\xi \quad \text{for} \quad r = r_c, \quad \lim_{r\to\infty}\frac{\partial P}{\partial r} = 0. \tag{1.12}$$

2. Differentiating both sides of Eqs. (1.4) and (1.7) with respect to t and eliminating $\partial^2 T/\partial t^2$, we obtain

$$\chi\frac{\partial}{\partial t}\nabla^2\left(P + \rho_2^0 W\right) - \frac{bd_2}{c_p}\frac{\partial}{\partial t}\nabla^2 T - \left(1 + \frac{b\alpha_2 T_0}{v}\right)\frac{\partial^2 P}{\partial t^2} - a\frac{\partial^2\Gamma_{hk}}{\partial t^2} = 0. \tag{2.1}$$

Substituting $\partial\nabla^2 T/\partial t$ from Eq. (2.1) in the equation obtained from Eq. (1.7) by taking the Laplace transform, we obtain

$$\chi\nabla^4\left(P + \rho_2^0 W\right) - \left(1 + \frac{\chi c_p}{d_2}\right)\frac{\partial}{\partial t}\nabla^2\left(P + \rho_2^0 W\right) + \frac{1}{d_2}\left(c_p + \alpha_2 b T_0\right)\frac{\partial^2 P}{\partial t^2} - a\left(\frac{\partial\nabla^2\Gamma_{hk}}{\partial t} - \frac{c_p}{d_2}\frac{\partial^2\Gamma_{hk}}{\partial t^2}\right) + \rho_2^0\nabla^2\frac{\partial W}{\partial t} = 0. \tag{2.2}$$

The variable P is replaced by the new function $\mathscr{P}$ and we then write the sum of the additional main stresses in the form

$$\Gamma_{kk}=f(t)+\frac{\chi\rho_2^0}{a}\int_0^t \nabla^2 W d\tau,\quad \mathscr{P}=P+\frac{ac_p}{c_p+\alpha_2 bT_0}f(t). \tag{2.3}$$

Then Eq. (2.2) will have the form

$$\chi\nabla^4\mathscr{P}-\left(1+\frac{\chi c_p}{d_2}\right)\nabla^2\frac{\partial\mathscr{P}}{\partial t}+\frac{1}{d_2}(c_p+\alpha_2 bT_0)\frac{\partial^2\mathscr{P}}{\partial t^2}=0. \tag{2.4}$$

The solution of Eq. (2.4) can be written in the form

$$\mathscr{P}=\mathscr{P}_1+\mathscr{P}_2,$$

where the functions $\mathscr{P}_1$ and $\mathscr{P}_2$ are solutions of problems obtained from Eqs. (1.12), (2.4):

$$\nabla^2\mathscr{P}_1+k_1^2\frac{\partial\mathscr{P}_1}{\partial t}=0; \tag{2.5}$$

$$\mathscr{P}_1=\gamma\xi+\frac{ac_p}{c_p+\alpha_2 bT_0}f(t)=\eta(t),\ r=r_c,\quad \lim_{r\to\infty}\frac{\partial\mathscr{P}_1}{\partial r}=0; \tag{2.6}$$

$$\nabla^2\mathscr{P}_2+k_2^2\frac{\partial\mathscr{P}_2}{\partial t}=0; \tag{2.7}$$

$$\mathscr{P}_2=0,\quad r=r_c,\quad \lim_{r\to\infty}\frac{\partial\mathscr{P}_2}{\partial r}=0. \tag{2.8}$$

In Eqs. (2.5), (2.7) the parameters k_1 and k_2 are roots of the algebraic equation

$$\chi k^4+\left(1+\frac{\chi c_p}{d_2}\right)k^2+\frac{1}{d_2}(c_p+\alpha_2 bT_0)=0. \tag{2.9}$$

The solution of Eqs. (2.5), (2.6) has the form [7]

$$\mathscr{P}_1=\int_{t_0}^t\frac{\partial U(r,t-\tau)}{\partial t}\eta(\tau)\,d\tau, \tag{2.10}$$

where

$$U(r,t)=1+\frac{2}{\pi}\int_0^\infty \exp\left[\left(\frac{z}{k_1}\right)^2 t\right]\frac{F(rz)\,zdz}{J_0^2(r_c z)+Y_0^2(r_c z)},$$

$$F(r,z)=J_0(rz)Y_0(r_c z)-J_0(r_c z)Y_0(rz)$$

is a solution of Eq. (2.5) which satisfies the condition $U(r, t_0)=0$, $U(r_c, t)=0$. Here J_0 and Y_0 are zeroth-order Bessel functions of the first and second sorts.

The solution to the second problem of Eqs. (2.7), (2.8) is identically equal to zero $\mathscr{P}_2\equiv 0$.

Substituting Eq. (2.3) in system (1.4), (1.7) and using Eq. (2.9) and solution (2.10), as well as the fact of temperature finiteness at infinity, we obtain

$$bT=-(1+\chi k_1^2)\mathscr{P}-\frac{ab\alpha_2 T_0}{c_p+\alpha_2 bT_0}f(t)+C^*, \tag{2.11}$$

where C^* is an integration constant which does not affect the temperature field, so that we may set $C^*\equiv 0$.

With the aid of Eqs. (2.10), (2.11) condition (1.11) can be reduced to the form

$$\frac{d\xi}{dt}=-\frac{2M}{\pi s^2}\left(\int_0^\infty\frac{F(rz)\,zdz}{J_0^2(r_c z)+Y_0^2(r_c z)}\int_{t_0}^\infty \exp\left[-\left(\frac{z}{s}\right)^2(t-\tau)\right]\eta(\tau)\,d\tau\right)+\Phi(t),\quad r=r_c, \tag{2.12}$$

where

$$s^2=\frac{1}{2\chi d_2}\left(d_2+\chi c_p-\sqrt{(\chi c_p-d_2)^2-4\chi b\alpha_2 d_2 T_0}\right),$$

$$M = D - \frac{B}{b}(1-\chi s^2), \quad k_1^2 = -s^2.$$

The function sought, $\xi(t)$, is a solution of a linear integrodifferential equation of the form of Eq. (2.12). It is known that the perturbing forces are periodic and can be represented as the sum of harmonic functions:

$$\Phi(t) = C_0' + \sum_{n=1}^{\infty}\left(C_n' \cos n\omega t + D_n' \sin n\omega t\right),$$
$$f(t) = C_0 + \sum_{n=1}^{\infty}\left(C_n \cos n\omega t + D_n \sin n\omega t\right). \tag{2.13}$$

We will seek the liquid level perturbation in the form of a Fourier series:

$$\xi(t) = A_0 + \sum_{n=1}^{\infty}\left(A_n \cos n\omega t + B_n \sin n\omega t\right), \tag{2.14}$$

where the known C_0', C_n', D_n', C_0, C_n, D_n and unknown A_0, A_n, B_n are Fourier coefficients; ω is the circle frequency of the exciting forces. We note that the process of field perturbation is sufficiently removed from the initial point, so that the initial conditions do not affect the pressure and temperature field distributions at the moment of observation, i.e., we take $t_0 \to -\infty$. Substituting Eqs. (2.13), (2.14) in Eq. (2.12) and integrating the latter, after grouping terms we obtain a system of algebraic equations

$$M\gamma f_1'(\lambda_n r_c) A_n - \left[n\omega - M\gamma f_2'(\lambda_n r_c)\right] B_n = -\frac{ac_p M}{c_p + \alpha_2 b T_0}\left[D_n f_2'(\lambda_n r_c) + C_n f_1'(\lambda_n r_c)\right] \mp C_n',$$
$$\left[n\omega - M\gamma f_2'(\lambda_n r_c)\right] A_n + M\gamma f_1'(\lambda_n r_c) B_n = -\frac{ac_p M}{c_p + \alpha_2 b T_0}\left[D_n f_1'(\lambda_n r_c) - C_n f_2'(\lambda_n r_c)\right] - D_n', \tag{2.15}$$

where

$$f_1(\lambda_n r) = \frac{1}{\Delta}\left[\ker(\lambda_n r)\ker(\lambda_n r_c) + \mathrm{kei}(\lambda_n r)\,\mathrm{kei}(\lambda_n r_c)\right],$$
$$f_2(\lambda_n r) = \frac{1}{\Delta}\left[\mathrm{kei}(\lambda_n r)\ker(\lambda_n r_c) - \ker(\lambda_n r)\,\mathrm{kei}(\lambda_n r_c)\right], \quad f'(\lambda_n r) = \frac{df(\lambda_n r)}{dr},$$
$$\Delta = (\ker(\lambda_n r_c))^2 + (\mathrm{kei}(\lambda_n r_c))^2, \quad \lambda_n = s\sqrt{n\omega}.$$

To illustrate the use of this method, we will take

$$\Phi(t) = C_1' \cos\omega t, \quad f(t) = C_1 \cos\omega t + D_1 \sin\omega t = A\cos(\omega t - \varphi).$$

Then the solution of system (2.15) in dimensionless variables will be

$$\frac{\gamma A_1}{A\cos\varphi} = -\frac{1}{\Delta_1}\{\theta_2\theta_3\left[f_1'^2(\theta_1) + f_2'^2(\theta_1)\right] + \theta_1\theta_3\left[\mathrm{tg}\,\varphi\cdot f_1'(\theta_1) - f_2'(\theta_1)\right] + \theta\theta_2\theta_1^{-1} f_1'(\theta_1)\},$$
$$\frac{\gamma B_1}{A\cos\varphi} = \frac{1}{\Delta_1}\{-\theta_2\theta_3\,\mathrm{tg}\,\varphi\left[f_1'^2(\theta_1) + f_2'^2(\theta_1)\right] + \theta_1\theta_3\left[\mathrm{tg}\,\varphi\cdot f_2'(\theta_1) + f_1'(\theta_1)\right]$$
$$+ \theta\theta_1^{-1}\left[\theta_1 - \theta_2 f_2'(\theta_1)\right]\}, \quad \Delta_1 = \left[\theta_2 f_1'(\theta_1)\right]^2 + \left[\theta_1 - \theta_2 f_2'(\theta_1)\right]^2,$$
$$\theta_1 = s r_c\sqrt{\omega}, \quad \theta_2 = M\gamma s^2 r_c, \quad \theta_3 = \frac{ac_p\theta_2}{c_p + \alpha_2 b T_0}, \quad \theta = \frac{C_1'\gamma s^2 r_c^2}{C_1}.$$

Using Eqs. (2.14), (2.3), (2.10), (2.11), and (1.8), we determine the liquid level, pressure, and temperature perturbations in the well

$$\xi(t) = A_*\cos(\omega t - \psi), \quad P = \gamma A_*\cos(\omega t - \psi),$$
$$T_c = \left(\frac{\beta_2}{\alpha_2} - \frac{1-\chi s^2}{b}\right)\gamma A_*\cos(\omega t - \psi) - \frac{(1-\chi s^2)\,a d_2 s^2}{b(c_p + \alpha_2 b T_0)} A\cos(\omega t - \varphi),$$

where

$$A_* = \sqrt{A_1^2 + B_1^2}, \quad \psi = \mathrm{arctg}(B_1/A_1), \quad T_c = T_1 + T.$$

The calculated stratum relaxation time $t_R = (sr_c)^2 \approx \frac{\mu\beta r_c^2}{k_0}\left(1 + \frac{\alpha_2 b T_0}{c_p}\right)$ depends not only on the piezoconductivity $\chi = k_0/\mu\beta$ and radius of the well, but also on the thermal parameters of the

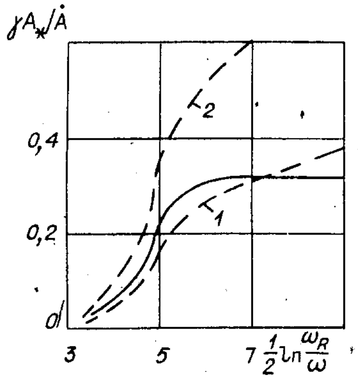

Fig. 1

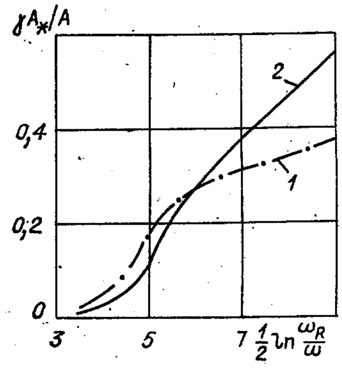

Fig. 2

saturated stratum, which at $T_0 > 0$ ($\alpha_2 > \alpha_1$) leads to a progressive process of field perturbation. Analysis reveals that due to thermal conductivity long-wave periodic heat propagation from the depths of the saturated water stratum into the well is insignificant. However, during the period of an earthquake the pressure $P = \gamma\xi(t)$ is perturbed by an amount (1-10) kg/cm^2, as a result of which the liquid temperature varies over a range of several degrees. For water this value in the well is equal to 0.3-3°C.

It is evident from Figs. 1 and 2 that the presence of gravitational potentials in the liquid filtration equation [see Eq. (1.3)] has an effect on liquid level variations in the well. If the direction of the pore pressure gradients and the gravitational potential coincide, an additional perturbation develops in the stratum, i.e., the well output, liquid level, and temperature fluctuations increase. Figure 1 shows the amplification of a signal A_*/A in water as a function of dimensionless frequency ω_R/ω for $\varphi = 1.0472$. The solid line corresponds to $C'/C_1 = 0$, with dashed lines 1, $C'/C_1 = 0.01$; 2, $C'/C_1 = -0.01$. It is evident from Fig. 2 that variations in water level in the well depend on the phase of the incident wave φ; curves 1, 2 correspond to $\varphi = 1.0472$; 0.3585.

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LITERATURE CITED

1. M. A. Sadovskii, F. I. Monakhov, and A. N. Semenov, "Hydrodynamic precursors of South Kuril earthquakes," Dokl. Akad. Nauk SSSR, 236, No. 1 (1977).
2. Yu. A. Kosygin, K. F. Sergeev, et al., "Lithospheric block interactions on the Central Sakhalin Fault, April 7-13, 1979," Dokl. Akad. Nauk SSSR, 254, No. 3 (1979).
3. S. S. Sardarov, "Tidal deformation of collectors," Dokl. Akad. Nauk SSSR, 243, No. 2 (1978).
4. A. Sterling and E. Smets, "Study of earth tides, earthquakes, and terrestrial spectroscopy by analysis of the level fluctuations in a borehole at Heibaart (Belgium)," Geophys. J. R. Astron. Soc., 23, 225 (1971).
5. I. G. Kissin, Earthquates and Subterranean Water [in Russian], Nauka, Moscow (1982).
6. V. N. Nikolaevskii, K. S. Basinev, A. T. Gorbunov, and G. A. Zotov, Mechanics of Saturated Porous Media [in Russian], Nedra, Moscow (1970).
7. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford Univ. Press (1959).